

# Scaling Effects in the Flexural Response and Failure of Composite Beams

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An experimental program was conducted in which scale model graphite-epoxy composite beams were loaded in bending until failure to investigate the effects of specimen size on flexural response and strength. Tests were performed on unidirectional, angle-ply, cross-ply, and quasi-isotropic beams of eight different scaled sizes ranging from 1/6 scale to full scale. The beams were subjected to an eccentric axial compressive load designed to promote large bending deformations and rotations. Results indicated that, although normalized flexural response was independent of specimen size, a significant scale effect in strength was observed for all laminate types. Typically, failure stresses and strains decreased as the size of the beam specimen increased from subscale to the full-scale prototype. Standard failure criteria for composite materials, such as the maximum stress, maximum strain, and tensor polynomial criteria, cannot predict the strength degradation with increasing specimen size. Consequently, a Weibull statistical model and a fracture mechanics model were used to analyze the strength scale effect.

## Introduction

SCALE model technology represents one method of investigating the behavior of advanced, weight-efficient composite structures under a variety of loading conditions. Testing of scale models can provide a cost-effective alternative to destructive testing of expensive composite prototypes and can be used to verify predictions obtained from analytical solutions and finite element models. However, the limitations involved in testing scale model structures must be understood before the technique can be fully utilized. Often test data obtained in the laboratory on small coupon-type specimens are "scaled up" to predict the response of full-scale structures with no regard for possible distortions due to specimen size or scale. This assumption is made even though a size effect in failure behavior of metallic<sup>1-4</sup> and advanced composite<sup>5-8</sup> structures has been documented.

Several researchers have investigated scaling effects in the response and failure of advanced fiber-reinforced composite structures. In general, the linear elastic response of composite laminates is independent of scaled size. However, a significant scale effect in the tensile strength of composite beams was observed by Kellas and Morton.<sup>5</sup> Similar results were found in the flexural strength of composite beams by Jackson.<sup>6</sup> Additionally, studies on scaling effects in the impact response and failure of composite beams by Morton<sup>7</sup> and composite plates by Qian et al.<sup>8</sup> demonstrate that the dynamic, elastic response behaves according to classical scaling laws. However, once the laminates are damaged, the response becomes more complex. Typically, smaller scale model specimens fail at higher stress and strain levels than their full-scale counterparts. The magnitude of the strength scale effect is dependent on the laminate stacking sequence and mode of failure. Standard failure criteria for composite materials, including the maximum stress, maximum strain, and tensor polynomial criteria, cannot pre-

dict the observed scale effect in strength when applied in a conventional manner. Various statistical theories and fracture mechanics approaches have been proposed to predict the strength scale effect since these techniques incorporate some measure of the absolute size of the test specimen.

An experimental and analytical study was conducted to characterize scaling effects in the large deflection response and failure of composite beams subjected to an eccentric axial compressive load. This loading configuration, shown schematically in Fig. 1, was chosen because it produces large bending deformations and promotes global failure of the beams away from the supported ends. A dimensional analysis was performed on the beam-column system using the methods outlined in Baker et al.<sup>9</sup> to determine the nondimensional parameters or Pi terms that govern the scaled response. Experiments were conducted to verify the scaling laws and to identify any deviations from scaled behavior. A one-dimensional large rotation "exact" beam analysis was derived to compare with the load-deflection response data and to predict failure loads. Two failure models, one based on Weibull statistics and one based on fracture mechanics theory, were applied to predict the observed degradation of strength with increasing specimen size.

## Experimental Program

Eight different sizes of beams including 1/6, 1/4, 1/3, 1/2, 2/3, 3/4, 5/6, and full scale were constructed of high modulus graphite-epoxy (AS4/3502) composite pre-preg material. Typical properties for this composite system were determined from material characterization tests<sup>10</sup> and are reported<sup>11</sup> to be  $E_1 = 136.79$  GPa (19.85 Mpsi),  $E_2 = 9.85$  GPa (1.43 Mpsi),  $G_{12} = 4.82$  GPa (0.70 Mpsi), and  $\nu_{12} = 0.293$ . Laminate stacking sequences including unidirectional, angle-ply, cross-ply, and quasi-isotropic were tested to examine a diversity of composite response and failure modes. The dimensions and lay-

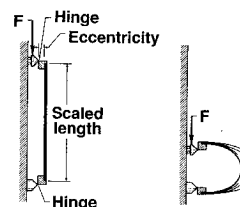


Fig. 1 Schematic drawing of the test setup.

Presented as Paper 91-1025 at the AIAA/ASME/ASCE/AHS/ASC 32nd Structures, Structural Dynamics, and Materials Conference, Baltimore, MD, April 8-10, 1991; received May 28, 1991; revision received Nov. 19, 1991; accepted for publication Nov. 19, 1991. Copyright © 1991 by the American Institute of Aeronautics and Astronautics, Inc. No copyright is asserted in the United States under Title 17, U.S. Code. The U.S. Government has a royalty-free license to exercise all rights under the copyright claimed herein for Governmental purposes. All other rights are reserved by the copyright owner.

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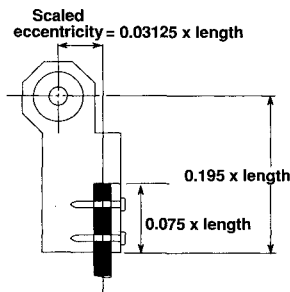


Fig. 2 Schematic drawing of the hinge fixture.

ups of each of the scale model beams are listed in Table 1. The thickness dimension of the beams was scaled by increasing the number of layers for each angular orientation in the laminate stacking sequence by the appropriate amount. This construction technique achieves scaled laminates on a "ply" level and insures that the in-plane and bending stiffnesses are scaled properly. However, using this approach, it is not possible to construct a 1/4 or 3/4 scale quasi-isotropic beam.

The basic loading configuration for the scaled beams is shown in the schematic drawing of Fig. 1. Each beam specimen was gripped in a set of hinges that offset the axial load with a moderate eccentricity. A detailed drawing of the hinge and beam attachment is shown in Fig. 2. Eight sets of hinges were constructed (one for each of the eight scale factors) to insure that the boundary and loading conditions were properly scaled. For each hinge, the eccentricity, grip length, and vertical distance from the center of the pin to the gauge portion of the beam were scaled.

The hinges were attached to the platens of a standard load test machine that applied a compressive vertical load. The hinged end fixture allowed the beam to undergo large rotations during deformation. Beam specimens were loaded in this manner until catastrophic failure, which is defined as complete loss of load-carrying capability. The beam-column loading configuration was chosen, in part, because failures occur in a global fashion at the center of the beam where the maximum bending moment occurs. Thus, failures are not introduced by local effects at the grip supports. Although the beam was loaded in a beam-column manner, the bending strains were two orders of magnitude greater than those due to the axial compressive load. Therefore, the beam was essentially loaded in a state of pure bending until first ply failure.

The vertical applied load was measured from a load platform located at the base of the bottom hinge support that was attached to the lower platen of the test machine. The distance traveled by the platens of the load test machine during a test is defined as the end displacement for that test. End displacement was measured using a string potentiometer displacement transducer. Strain measurements were recorded from back-to-back gauges applied at the midpoint of the beams. Additional information on the experimental procedure used for the flexural tests is reported in Ref. 6.

### Analysis

A one-dimensional, large rotation exact solution was developed to predict the response of the composite beam-column under eccentric axial load. The governing equation was derived from equilibrium of the forces and moments on a beam element. The exact moment and curvature relation was used in the analysis, thus allowing the solution to predict large rotation response. The solution of the governing equation follows the development outlined in Timoshenko and Gere<sup>12</sup> for the classical "elastica" problem of a tip-loaded cantilever beam but was adapted for the beam-column problem by applying the end moment boundary conditions produced by the eccentric vertical load. The beam analysis predicts the end displacement, transverse midpoint displacement, and end rotation as a function of applied end load. A stress analysis was

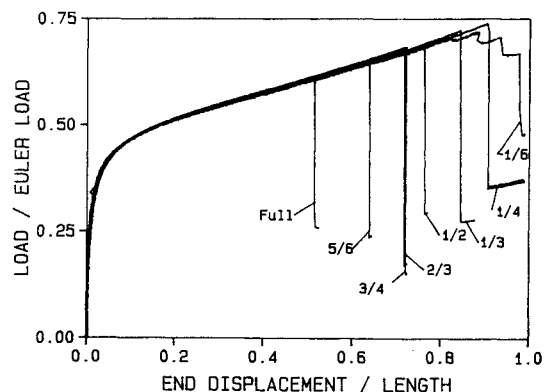
incorporated to predict the individual ply stresses for each load step so that stress-based failure criteria could be applied. Complete details of the solution derivation and implementation are provided in Ref. 6.

## Results

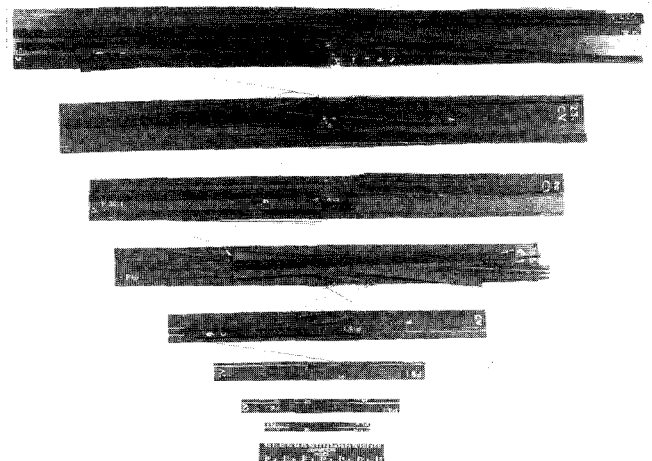
### Normalized Load-Deflection Response and Failure Mechanisms

Normalized load vs end displacement plots and corresponding photographs of a complete (1/6 through full scale) set of failed beam specimens for the unidirectional, angle-ply, cross-ply, and quasi-isotropic laminates are shown in Figs. 3-6, respectively. Vertical load was normalized by the Euler column buckling load that was determined empirically for each beam specimen. End displacement was normalized by the gauge length of the beam. The load response curves for the unidirectional and cross-ply laminate families tend to fall along a single curve, as shown in Figs. 3a and 5a, which indicates that no scaling effects are observed in the static load response, even for very large displacements and rotations. However, the angle-ply and quasi-isotropic laminates deviated from scaled response due to damage events that altered the beam stiffness. Thus, the success of achieving scaled response is dependent on the laminate stacking sequence with the best results seen for laminates with a large percentage of 0-deg plies.

The photographs shown in Figs. 3b-6b indicate that, although the failure modes for the laminate types considered in this study are different from each other, they are similar between scaled beams within a laminate family. Thus, failure mechanisms appear to be independent of specimen size. The unidirectional beams, shown in Fig. 3b, failed by fiber fracture near the midpoint of the beam. This failure mode is



a) Normalized load vs end displacement

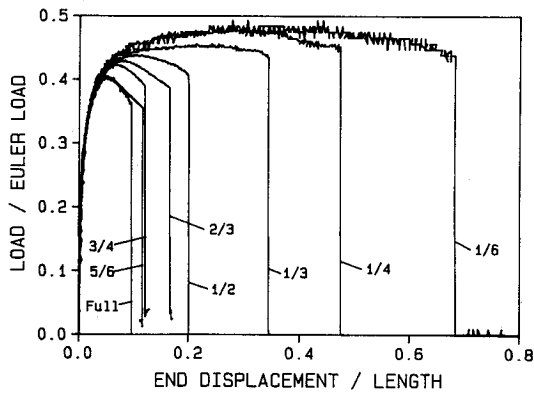


b) Failed beam specimens

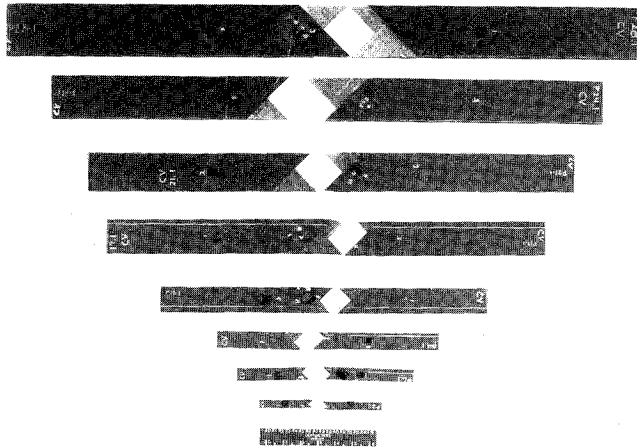
Fig. 3 Unidirectional scale model beam results.

Table 1 Scale model beam dimensions and stacking sequences

Scale factor	Beam dimension, mm $\times$ mm (in. $\times$ in.)	Unidirectional	Angle ply	Cross ply	Quasi-isotropic
1/6	12.7 $\times$ 127 (0.5 $\times$ 5.0)	[0] <sub>8T</sub>	[45 <sub>2</sub> / - 45 <sub>2</sub> ] <sub>S</sub>	[0 <sub>2</sub> /90 <sub>2</sub> ] <sub>S</sub>	[ - 45/0/45/90] <sub>S</sub>
1/4	19.05 $\times$ 190.5 (0.75 $\times$ 7.5)	[0] <sub>12T</sub>	[45 <sub>3</sub> / - 45 <sub>3</sub> ] <sub>S</sub>	[0 <sub>3</sub> /90 <sub>3</sub> ] <sub>S</sub>	—
1/3	25.4 $\times$ 254 (1.0 $\times$ 10.0)	[0] <sub>16T</sub>	[45 <sub>4</sub> / - 45 <sub>4</sub> ] <sub>S</sub>	[0 <sub>4</sub> /90 <sub>4</sub> ] <sub>S</sub>	[ - 45 <sub>2</sub> /0 <sub>2</sub> /45 <sub>2</sub> /90 <sub>2</sub> ] <sub>S</sub>
1/2	38.1 $\times$ 381 (1.5 $\times$ 15.0)	[0] <sub>24T</sub>	[45 <sub>6</sub> / - 45 <sub>6</sub> ] <sub>S</sub>	[0 <sub>6</sub> /90 <sub>6</sub> ] <sub>S</sub>	[ - 45 <sub>3</sub> /0 <sub>3</sub> /45 <sub>3</sub> /90 <sub>3</sub> ] <sub>S</sub>
2/3	50.8 $\times$ 508 (2.0 $\times$ 20.0)	[0] <sub>32T</sub>	[45 <sub>8</sub> / - 45 <sub>8</sub> ] <sub>S</sub>	[0 <sub>8</sub> /90 <sub>8</sub> ] <sub>S</sub>	[ - 45 <sub>4</sub> /0 <sub>4</sub> /45 <sub>4</sub> /90 <sub>4</sub> ] <sub>S</sub>
3/4	57.15 $\times$ 571.5 (2.25 $\times$ 22.5)	[0] <sub>36T</sub>	[45 <sub>9</sub> / - 45 <sub>9</sub> ] <sub>S</sub>	[0 <sub>9</sub> /90 <sub>9</sub> ] <sub>S</sub>	—
5/6	63.5 $\times$ 635 (2.5 $\times$ 25.0)	[0] <sub>40T</sub>	[45 <sub>10</sub> / - 45 <sub>10</sub> ] <sub>S</sub>	[0 <sub>10</sub> /90 <sub>10</sub> ] <sub>S</sub>	[ - 45 <sub>5</sub> /0 <sub>5</sub> /45 <sub>5</sub> /90 <sub>5</sub> ] <sub>S</sub>
6/6	76.2 $\times$ 762 (3.0 $\times$ 30.0)	[0] <sub>48T</sub>	[45 <sub>12</sub> / - 45 <sub>12</sub> ] <sub>S</sub>	[0 <sub>12</sub> /90 <sub>12</sub> ] <sub>S</sub>	[ - 45 <sub>6</sub> /0 <sub>6</sub> /45 <sub>6</sub> /90 <sub>6</sub> ] <sub>S</sub>

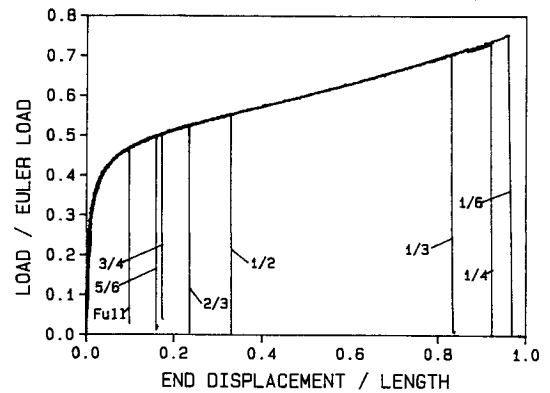


a) Normalized load vs end displacement

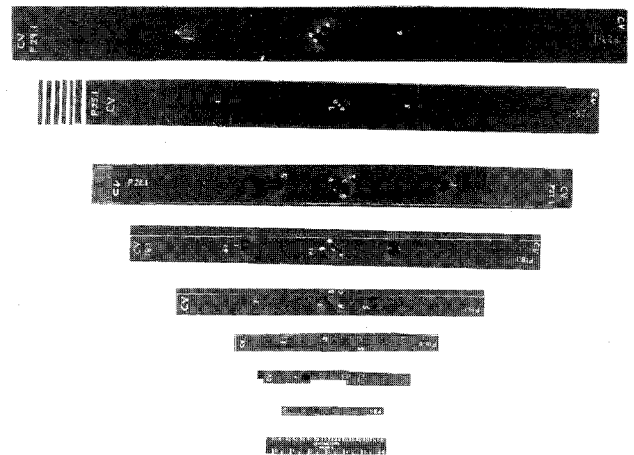


b) Failed beam specimens

Fig. 4 Angle-ply scale model beam results.



a) Normalized load vs end displacement



b) Failed beam specimens

Fig. 5 Cross-ply scale model beam results.

typical of all of the unidirectional beams 1/6 through full scale. Failure of the angle-ply beams occurred by transverse matrix cracking along the 45-deg fiber line. There was no evidence of fiber breakage, as shown in Fig. 4b, for any of the angle-ply beams.

The cross-ply laminates exhibited a transition in the mode of failure with specimen size. In general, the cross-ply laminates failed by transverse matrix cracking in the 90-deg core accompanied by either a delamination along the 0- and 90-deg ply interface or fiber breakage in the 0-deg plies. The smaller scale model beams contained some fiber fractures, whereas the larger scale model beams failed by delamination, as shown in

Fig. 5b. The transition occurred between the 1/3 and 1/2 scale model beams and is associated with the large gap in the normalized end displacement ratios at failure for these beams, as shown in the load-deflection plot of Fig. 5a. During large deformations, the 90-deg plies located in the center of the cross-ply laminates developed transverse matrix cracks. The cracks were evenly spaced and resulted in uniform pieces of debris, some of which are shown in Fig. 5b for the 5/6 scale beam.

The quasi-isotropic beams failed through a combination of matrix cracking, delamination, and some fiber failure. Although the photograph in Fig. 6b does not give a good indica-

**Table 2** Average failure loads, displacements, and strains for scaled beams

Scale factor	Failure load, N, (lb)		Failure load ratio	End disp. ratio	Failure strain, %	
					Tension	Comp.
Unidirectional						
1/6	77.4	(17.4)	0.67	0.98	— <sup>a</sup>	—
1/4	186.8	(42.0)	0.73	0.91	1.72	− 2.01
1/3	278.4	(62.6)	0.72	0.86	1.56	− 1.82
1/2	633.4	(142.4)	0.68	0.74	1.41	− 1.70
2/3	1209.0	(271.8)	0.69	0.75	1.35	− 1.67
3/4	1564.8	(351.8)	0.69	0.74	1.443	− 1.66
5/6	1807.7	(406.4)	0.67	0.68	1.35	− 1.54
Full	2331.3	(524.1)	0.62	0.54	1.13	− 1.24
Angle ply						
1/6	8.9	(2.0)	0.45	0.67	1.74	− 1.66
1/4	19.6	(4.4)	0.44	0.47	1.22	− 1.36
1/3	28.5	(6.4)	0.44	0.36	0.96	− 1.12
1/2	52.5	(11.8)	0.41	0.21	0.62	− 0.79
2/3	86.7	(19.5)	0.35	0.16	0.84	− 0.74
3/4	125.9	(28.3)	0.39	0.11	0.46	− 0.52
5/6	115.6	(26.0)	0.34	0.15	0.66	− 0.77
Full	166.8	(37.5)	0.35	0.11	0.55	− 0.46
Cross ply						
1/6	76.1	(17.1)	0.76	0.96	—	− 2.19
1/4	139.2	(31.3)	0.75	0.95	1.67	− 2.09
1/3	244.2	(54.9)	0.71	0.84	1.50	− 1.89
1/2	443.0	(99.6)	0.56	0.35	0.86	− 1.01
2/3	790.0	(177.6)	0.53	0.26	0.72	− 0.83
3/4	975.4	(219.3)	0.51	0.19	0.59	− 0.67
5/6	1222.8	(274.9)	0.50	0.16	0.54	− 0.61
Full	1535.0	(345.1)	0.47	0.10	0.39	− 0.42
Quasi-isotropic						
1/6	31.1	(7.0)	0.73	0.80	1.43	− 1.70
1/3	107.2	(24.1)	0.62	0.53	1.06	− 1.45
1/2	230.9	(51.9)	0.60	0.59	—	− 1.49
2/3	375.4	(84.4)	0.54	0.37	0.91	− 1.16
5/6	583.1	(131.1)	0.54	0.31	0.78	− 1.00
Full	833.1	(187.3)	0.53	0.29	0.69	− 0.95

<sup>a</sup>Gauge failed.

tion, the damaged quasi-isotropic beams are highly curved. The sequence of failure events occurred such that the remaining intact section of the beam consisted of an unsymmetric laminate, resulting in the observed curvature. Although the smaller scale model beams were more extensively damaged than the larger beams, the mechanism of failure was similar for all beam sizes.

### Strength

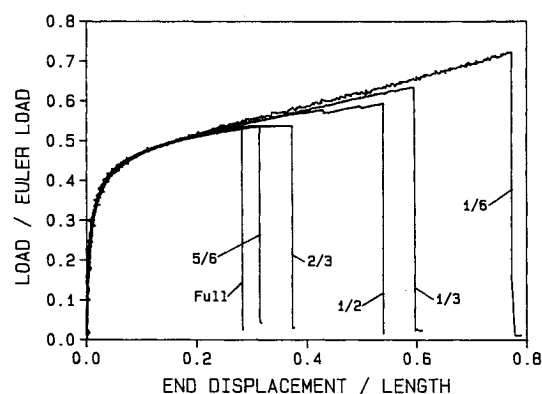
A significant scale effect in strength is observed for all of the laminate families. Figures 3-6 show that the normalized loads and end displacements at failure increase as the size of the specimen decreases from the full-scale prototype to the 1/6 scale model. Tensile and compressive strains at failure indicate a similar trend. Table 2 lists the loads, load ratios, end displacement ratios, and tensile and compressive strains at failure for the unidirectional, angle-ply, cross-ply, and quasi-isotropic laminates, respectively. The scale effect in strength is particularly large for the cross-ply laminates in which the 1/6 scale beam failed at a normalized load 1.5 times the value for the full-scale beam and at an end displacement ratio 10 times greater than the full-scale beam.

The exact large deflection beam solution was used to perform a stress analysis of the eccentrically loaded composite beam column. Three standard failure criteria including the maximum stress, maximum strain, and Tsai-Wu tensor polynomial criteria were applied to predict first ply failure. The criteria require that five material strength properties be known, including tensile fiber-direction strength  $X_t$ , compressive fiber-direction strength  $X_c$ , tensile transverse strength  $Y_t$ , compressive transverse strength  $Y_c$ , and in-plane shear

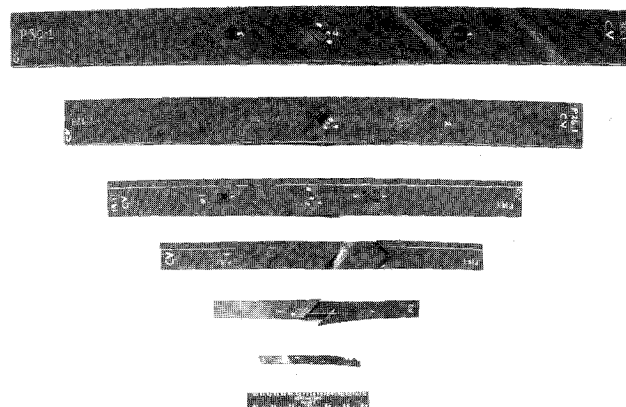
strength  $S$ . The values of failure strain are found by dividing the strengths by the corresponding moduli. The strength values were determined for the AS4/3502 graphite-epoxy material system by Sensmeier et al.<sup>10</sup> from a series of material characterization tests. Results of the failure analysis are shown in Figs. 7-10 for the unidirectional, angle-ply, cross-ply, and quasi-isotropic beams, respectively. These plots indicate the load and end displacement ratios at which first ply failure is predicted, along with the experimental load-deflection data from a full-scale beam and a smaller scale model beam (either 1/6 or 1/4 scale). The sudden drop in load for the experimental data indicates the load ratio at which ultimate failure occurred.

Results of the failure analysis for the unidirectional laminates indicate that failure occurs at the midspan of the beam on the compression side at a load ratio of 0.51 and an end displacement ratio of 0.18, as shown in Fig. 7. All three failure criteria predict that failure will occur at that load and end displacement ratio. The maximum stress and strain criteria predict that the compressive stress and strain in the fiber direction exceeds the compressive strength and ultimate compressive strain in the 0-deg ply on the outer surface. The plot of Fig. 8 shows the normalized load-displacement response up to failure for the angle-ply laminate as predicted from the beam analysis, along with experimental data from a 1/6 and full-scale beam. The Tsai-Wu criterion predicts that the first ply failure will occur in the outer 45-deg ply on the tensile side of the beam at a load ratio of 0.52 and an end displacement ratio of 0.2. The maximum stress and maximum strain criteria predict failure at a higher load ratio of 0.57 and an end displacement ratio of 0.36. Both of these criteria predict a shear failure of the outer 45-deg ply on the compression side of the beam.

For the cross-ply laminates, the maximum stress and maximum strain criteria predict first ply failure to occur at a load



a) Normalized load vs end displacement



b) Failed beam specimens

**Fig. 6** Quasi-isotropic scale model beam results.

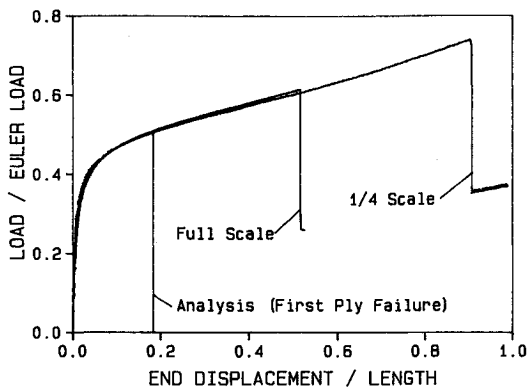


Fig. 7 Large deflection analysis including predicted first ply failure for unidirectional laminates. Experimental load-deflection response from 1/4 and full-scale beams.

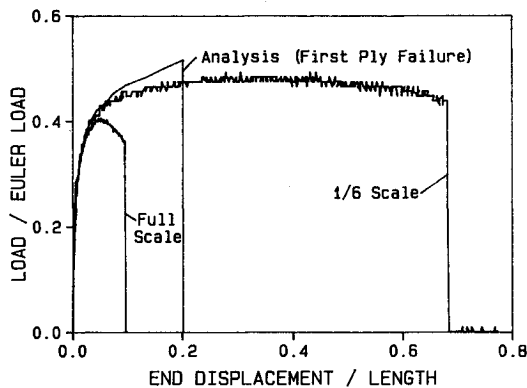


Fig. 8 Large deflection analysis including predicted first ply failure for angle-ply laminates. Experimental load-deflection response from 1/6 and full-scale beams.

ratio of 0.53 and an end displacement ratio of 0.24, as indicated in Fig. 9. A compressive failure is predicted in the outer 0-deg ply of the beam. The Tsai-Wu criterion is exceeded at a higher load ratio of 0.55 and an end displacement ratio of 0.29 in the outer 0-deg ply on the compressive side of the beam. For the quasi-isotropic laminates, the Tsai-Wu failure criterion predicts that first ply failure occurs in the 0-deg ply in compression at a load ratio of 0.52 and an end displacement ratio of 0.2. This failure location is shown in Fig. 10 along with the load-displacement response for a 1/6 and full-scale quasi-isotropic beam. The maximum stress and maximum strain criteria also predict that the first ply failure will occur in the 0-deg ply on the compressive side of the beam, but at a higher load ratio (0.55) and end displacement ratio (0.3).

For the angle-ply and cross-ply laminates, the analytical prediction of first ply failure falls between the observed failures for the full-scale and subscale beams. For the unidirectional and quasi-isotropic laminates, the analytical failure predictions are conservative, even for the full-scale specimens. Obviously, the failure criteria do not predict a difference in strength based on specimen size. In part this is because elementary approaches to scaling require that stress and strain scale as unity. Consequently, for geometrically similar beams, any failure criteria based solely on stress or strain will predict a single failure load ratio, independent of the absolute specimen size.

#### Analysis of the Strength Scale Effect

Previous researchers have attempted to model the strength scale effect of composite structures using a Weibull statistical model<sup>13-16</sup> or a fracture mechanics based model.<sup>4,17</sup> The application of statistical techniques for modeling the size effect in strength of brittle materials is based on the observation that

these materials are flaw sensitive. Since the presence of imperfections can be statistical in nature, it is reasonable to assume that larger specimens will exhibit a lower strength simply because the probability is higher that a strength critical flaw, such as a crack or void, is present in the greater volume of material. Weibull statistical theory has been applied in conjunction with a weakest link theory to develop a mathematical model for predicting the scale effect in strength. Bullock<sup>14</sup> found that the ratio of ultimate strengths between a geometrically similar model and prototype is given by

$$\frac{S_m^{\text{ult}}}{S_p^{\text{ult}}} = \left[ \frac{V_p}{V_m} \right]^{1/\beta}$$

where the subscripts *m* and *p* refer to the model and prototype, respectively; *S* is the ultimate stress, *V* the volume, and  $\beta$  the Weibull shape parameter, sometimes called the flaw density parameter since it provides a measure of the scatter in the strength data. The flaw density parameter  $\beta$  is assumed to be a material constant. If  $\beta$  is determined empirically from two specimens of differing size, then the strength of geometrically similar scale models can be predicted.

Atkins and Caddell<sup>4</sup> used a fracture mechanics approach to derive a simple size-strength relationship for notched brittle materials. They included the critical stress intensity factor in a dimensional analysis and, based on the laws of similitude, showed that the stress for unstable crack growth should scale as  $\lambda^{-1/2}$ . Thus, the stress needed to propagate a crack in a full-scale prototype structure  $\sigma_p$  and the corresponding stress  $\sigma_m$  in a model structure are related by

$$\sigma_p = \sigma_m \sqrt{\lambda}$$

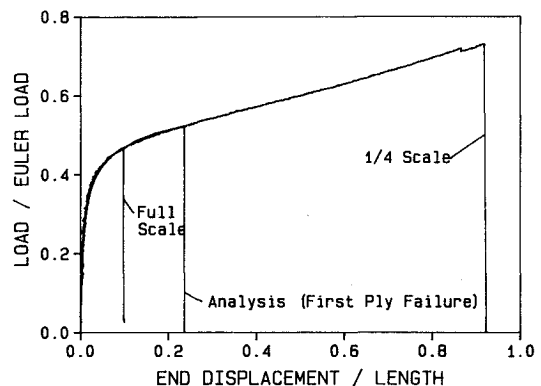


Fig. 9 Large deflection analysis including predicted first ply failure for cross-ply laminates. Experimental load-deflection response from 1/4 and full-scale beams.

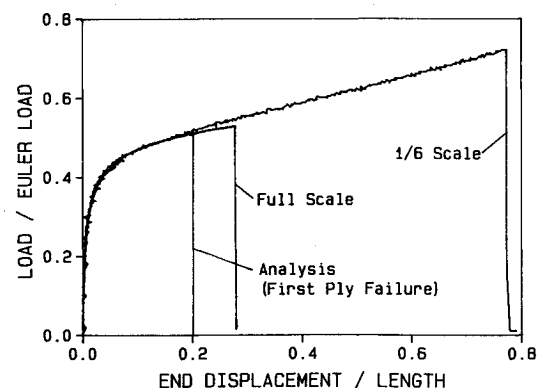


Fig. 10 Large deflection analysis including predicted first ply failure for quasi-isotropic laminates. Experimental load-deflection response from 1/6 and full-scale beams.

where  $\lambda$  is the geometric scale factor that is defined as the ratio of the length of the model to the length of the prototype. For this investigation,  $\lambda$  is equal to the scale factors 1/6, 1/4, 1/3, 1/2, 2/3, 3/4, 5/6, and 1.

The normalized load ratio at failure is plotted as a function of scale factor for the unidirectional, angle-ply, cross-ply, and quasi-isotropic laminates, respectively, in Figs. 11-14. The data for these plots were taken from Table 2. The value of failure load ratio for each of the scale model beams was normalized by the full-scale value for that laminate family. If no scale effect in strength was present, then all of the data would fall on the straight lines drawn at 1.0 in each of the plots. Results from both the Weibull statistical model and the fracture mechanics based model are plotted with the experimental data. The flaw density parameter was determined from strength data for a small scale and full-scale size beam specimen for each laminate type. As shown in Figs. 11-14, the Weibull model shows better agreement with the strength data than does the fracture mechanics model. The success of the Weibull model in predicting strength degradation depends heavily on the flaw density parameter that must be determined empirically and varies depending on the laminate stacking sequence. As such, the flaw density parameter may be influenced by the initial damage state and choice of beam sizes used to derive it. Jackson and Morton<sup>11</sup> reported results that indi-

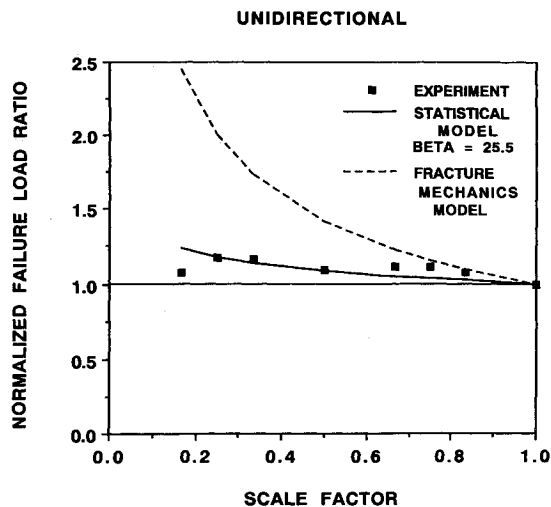


Fig. 11 Comparison of normalized failure load vs scale factor with statistical and fracture mechanics based analytical models for unidirectional scale model beams.

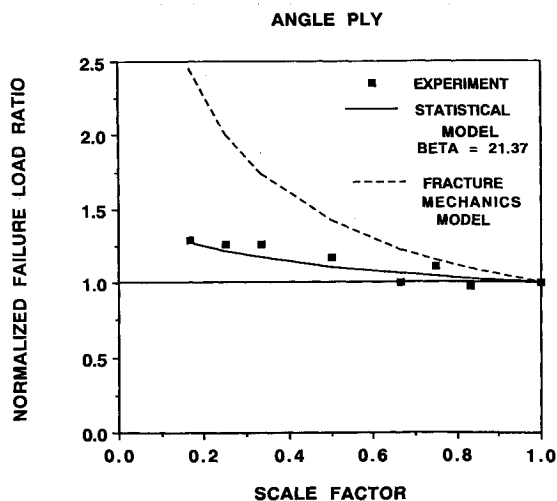


Fig. 12 Comparison of normalized failure load vs scale factor with statistical and fracture mechanics based analytical models for angle-ply scale model beams.

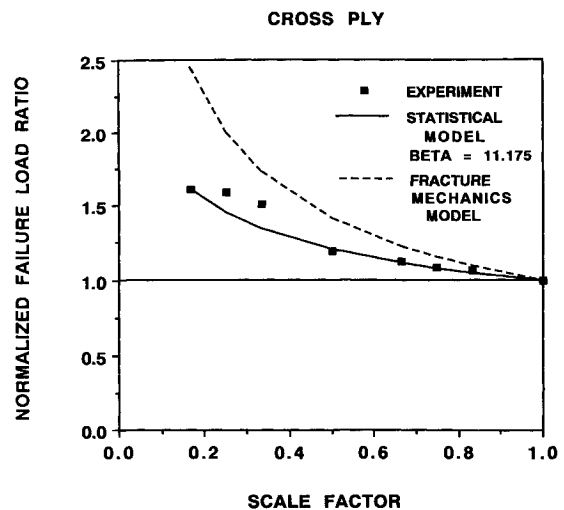


Fig. 13 Comparison of normalized failure load vs scale factor with statistical and fracture mechanics based analytical models for cross-ply scale model beams.

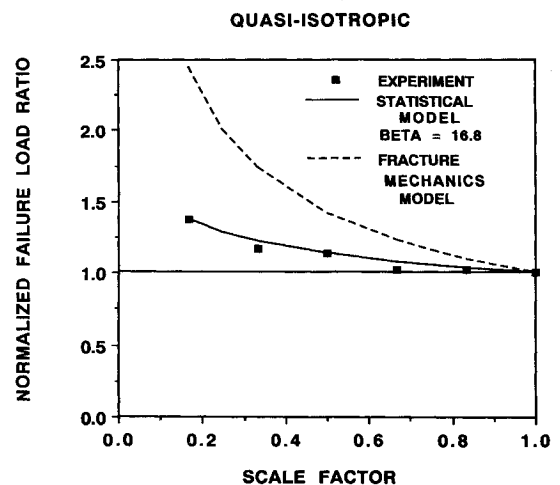


Fig. 14 Comparison of normalized failure load vs scale factor with statistical and fracture mechanics based analytical models for quasi-isotropic scale model beams.

cated that differences in the initial damage state and transitions in the mode of failure between the different beam sizes can lead to errors in the value of the flaw density parameter. In these cases, the Weibull model can overpredict the strength of the full-scale beam.

Results of applying statistical and fracture mechanics based models for predicting the strength effect show that neither approach can explain the phenomenon in a satisfactory manner. Research by Crossman et al.,<sup>18</sup> Wang et al.,<sup>19</sup> and Laws and Dvorak<sup>20</sup> on the effects of transverse matrix cracking on the final fracture of cross-ply laminates suggests that a model that incorporates both theories is needed. A statistical approach was used to determine which microflaws within a 90-deg core ply would most likely coalesce to form a transverse matrix crack given a random distribution of flaws and flaw sizes. Once a crack formed, fracture mechanics theories were applied to determine the stability of the crack under the given loading conditions. The progression of crack formation and stability were continually monitored until ultimate laminate failure. A model of this type was used to successfully predict the tensile failure of cross-ply laminates in which the number of 90-deg plies was varied from two to six. These laminates were not replica models since the number of 0-deg plies was not adjusted in the same proportion as the number of 90-deg plies; however, the success of the model indicates that

it may be able to predict accurately the scale effect in strength for laminates of varying sizes and stacking sequences.

In general, it is believed that the size effect in strength that is observed on a structural level is the result of microscopic damage that begins within the laminate and develops in a certain manner under the applied load. The accumulation of damage and interaction of failure mechanisms eventually produce ultimate failure of the structure. The effect of test specimen size on failure modes and ultimate strength needs to be understood on a material level before scale model testing of composite structures can be used to validate full-scale behavior.

### Summary

Scale model composite beams were loaded to failure under an eccentric axial compressive load to study the effects of absolute specimen size on the large deflection flexural response, ultimate strength, and mode of failure. Experiments were conducted on graphite-epoxy composite beams having unidirectional, angle-ply, cross-ply, and quasi-isotropic laminate stacking sequences. The beams were fabricated to insure geometric and constitutive similarity and were tested under scaled loading conditions. Results of this investigation show that valuable information can be obtained from testing scale model composite structures, especially in the linear elastic response region. However, a significant strength scale effect was observed experimentally for all laminates tested. Small-scale beams fail at higher normalized load and end displacement values than their full-scale counterparts. Failure theories for composites such as maximum stress, maximum strain, and Tsai-Wu tensor polynomial criteria cannot predict a difference in strength based on absolute specimen size. Weibull statistical approaches and fracture mechanics based models were applied to predict the observed scale effect in strength. The statistical model gave better agreement with experimental data, but an empirical flaw density parameter was necessary for good correlation. Additional research is required to isolate and understand the factors responsible for the size effect in failure of composite structures before meaningful models can be developed.

### References

- <sup>1</sup>Booth, E., Collier, D., and Miles, J., "Impact Scalability of Plated Steel Structures," *Structural Crashworthiness*, Butterworths, London, 1983, pp. 136-174.
- <sup>2</sup>Duffey, T. A., "Scaling Laws for Fuel Capsules Subjected to Blast, Impact, and Thermal Loading," *Proceedings of the 1971 Inter-society Energy Conversion Engineering Conference*, Boston, MA, Aug. 1971, pp. 775-786 (also Sandia Corp., N70-38879, Albuquerque, NM).
- <sup>3</sup>Jones, N., "Scaling of Inelastic Structures Loaded Dynamically," *Structural Impact and Crashworthiness*, edited by G. A. O. Davies, Vol. 1, Keynote Lectures, Elsevier Applied Science Publishers, London, 1984, pp. 46-71.
- <sup>4</sup>Atkins, A. G., and Caddell, R. M., "The Laws of Similitude and Crack Propagation," *International Journal of Mechanical Sciences*, Vol. 16, No. 8, 1974, pp. 541-548.
- <sup>5</sup>Kellas, S., and Morton, J., "Strength Scaling in Fiber Composites," NASA CR 4335, Nov. 1990.
- <sup>6</sup>Jackson, K. E., "Scaling Effects in the Static and Dynamic Response of Graphite-Epoxy Beam-Columns," NASA TM 102697, July 1990.
- <sup>7</sup>Morton, J., "Scaling of Impact-Loaded Carbon-Fiber Composites," *AIAA Journal*, Vol. 26, No. 8, 1988, pp. 989-994.
- <sup>8</sup>Qian, Y., Swanson, S. R., Nuismer, R. J., and Bucinell, R. B., "An Experimental Study of Scaling Rules for Impact Damage in Fiber Composites," *Journal of Composite Materials*, Vol. 24, No. 5, 1990, pp. 559-570.
- <sup>9</sup>Baker, W. E., Westine, P. S., and Dodge, F. T., *Similarity Methods in Engineering Dynamics*, Hayden Book, Rochelle Park, NJ, 1973, pp. 35-46.
- <sup>10</sup>Sensmeier, M. D., Griffin, O. H., and Johnson, E. R., "Static and Dynamic Large Deflection Flexural Response of Graphite-Epoxy Beams," NASA CR 4118, March 1988.
- <sup>11</sup>Jackson, K. E., and Morton, J., "Evaluation of Some Scale Effects in the Response and Failure of Composite Beams," *Proceedings of the First NASA Advanced Composites Technology Conference*, Seattle, WA, Oct. 29-Nov. 1, 1990, NASA CP 3104, Pt. 2, pp. 979-1003.
- <sup>12</sup>Timoshenko, S. P., and Gere, J. M., *Theory of Elastic Stability*, McGraw-Hill, New York, 1961, pp. 76-82.
- <sup>13</sup>Zweben, C., "The Effect of Stress Nonuniformity and Size on the Strength of Composite Materials," *Composites Technology Review*, Vol. 3, No. 1, 1981, pp. 23-26.
- <sup>14</sup>Bullock, R. E., "Strength Ratios of Composite Materials in Flexure and Tension," *Journal of Composite Materials*, Vol. 8, April 1974, pp. 200-206.
- <sup>15</sup>Wang, A. S. D., Tung, R. W., and Sanders, B. A., "Size Effect on Strength and Fatigue of a Short Fiber Composite Material," *Emerging Technologies in Aerospace Structures, Design, Structural Dynamics and Materials*, American Society of Mechanical Engineers, New York, Aug. 1980, pp. 37-52.
- <sup>16</sup>Chou, P. C., and Croman, R., "Scale Effect in Fatigue of Composite Materials," *Journal of Composite Materials*, Vol. 13, July 1979, pp. 178-194.
- <sup>17</sup>Carpinteri, A., and Bocca, P., "Transferability of Small Specimen Data to Full-Size Structural Components," *Composite Material Response: Constitutive Relations and Damage Mechanisms*, edited by G. C. Sih, G. F. Smith, I. H. Marshall, and J. J. Wu, Elsevier Applied Science, London, 1987, pp. 111-131.
- <sup>18</sup>Crossman, F. W., Warren, W. J., and Wang, A. S. D., "Influence of Ply Thickness on Damage Accumulation and Final Fracture," *Advances in Aerospace Structures, Materials, and Dynamics: A Symposium on Composites*, American Society of Mechanical Engineers, ASME Pub. AD-06, New York, 1983, pp. 215-226.
- <sup>19</sup>Wang, A. S. D., Chou, P. C., and Lei, S. C., "A Stochastic Model for the Growth of Matrix Cracks in Composite Laminates," *Advances in Aerospace Structures, Materials, and Dynamics: A Symposium on Composites*, American Society of Mechanical Engineers, ASME Pub. AD-06, New York, 1983, pp. 7-16.
- <sup>20</sup>Laws, N., and Dvorak, G. J., "Progressive Transverse Cracking in Composite Laminates," *Journal of Composite Materials*, Vol. 22, No. 10, 1988, pp. 900-916.